Karlsruhe
Aug 20, 2012
corrected Aug 21, 2012

Preliminary Proposal for Partitioning a Circular Disk into 5 Parts

Wolfdieter L a n g [1]

Motivated by Neil Sloane's seqfan e-mails from May 2012 which referred to the figure on p. 4 of the paper "Les partages d'un Polygone convexe en 4 Polygones Semblables au Premier" by G. Valette and T. Zamfirescu, J. Comb. Theory, (B) 16, (1974) 1-16, available under http://tzamfirescu.tricube.de , we propose a set of 92 partitions of a circular disk into $n = 5$ parts which satisfy the requirements given in that paper on p. 3, namely i) each part is topologically equivalent to a circular disk (*Jordan* domain), and the union of all the n parts with boundary is a circlular disk, ii) the pairwise intersections of the parts without boundary are empty, and iii) the pairwise intersections of the parts with boundary are connected.

For example, a part obtained by the intersection of two overlapping circles which touch each other in a point (a sickle-shaped part) is not allowed because this is topologically not a circular disk. Whenever two inner nodes are connected by two or more edges rule iii) will be violated. Each node has degree $d \geq 3$. *Euler*'s formula is $n = (e + b) - (i + b) + 1 = e - i + 1$, with internal nodes $Q_1, ..., Q_i$, internal edges $F_1, ..., F_e$ and boundary edges $E_1, ..., E_b$ (these are the names proposed by Neil Sloane). Twice the total number of edges is obtained from the sum over the degrees d of all nodes: $2\,(b + e) = \sum_{j=1}^{b} d(P_j) + \sum_{j=1}^{i} d(Q_j)$.

It may be that some partition is missing. Also, undetected multiple counting may be present. All examples should satisfy the rules.

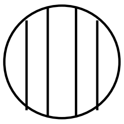
If 92 should be correct the sequence would start like $\{1, 1, 3, 15, 92, ...\}$ (probably not related to A020108).

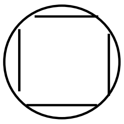
Error detection log:

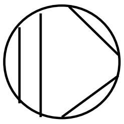
1. David Scambler, Aug 21 2012: old e) i) 1 = e) i) 2

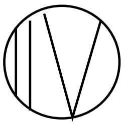
[1] wolfdieter.lang@kit.edu, http://www-itp.particle.uni-karlsruhe.de/~wl

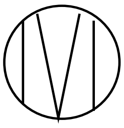
a) Number of internal nodes i = 0

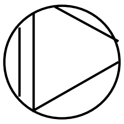

1

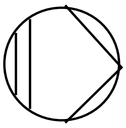

2

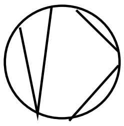

3

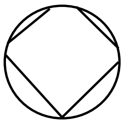

4

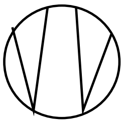

5

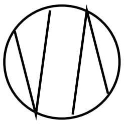

6

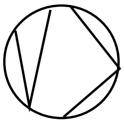

7

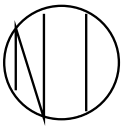

8

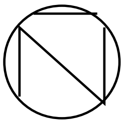

9

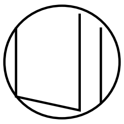

10

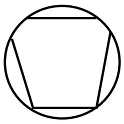

11

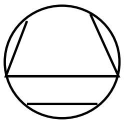

12

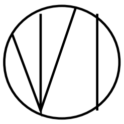

13

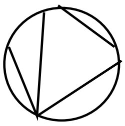

14

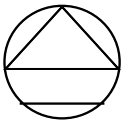

15

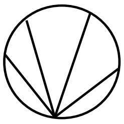

16

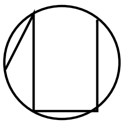

17

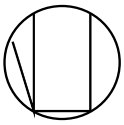

18

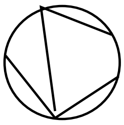

19

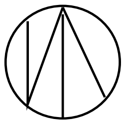

20

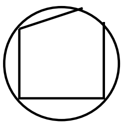

21

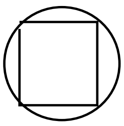

22

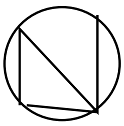

23

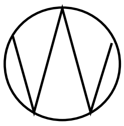

24


25


26


27

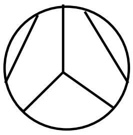

28


29

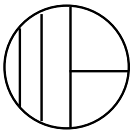
b) Number of internal nodes i = 1

i) degree 3 node

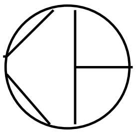
i)



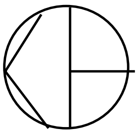
1



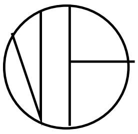
2



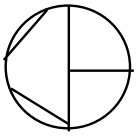
3



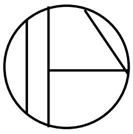
4



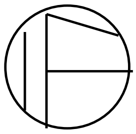
5



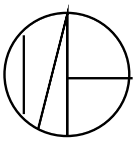
6



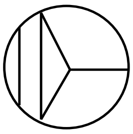
7



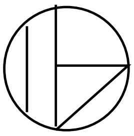
8



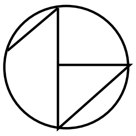
9



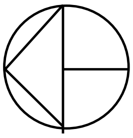
10



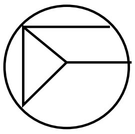
11



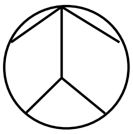
12



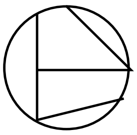
13



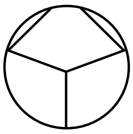
14



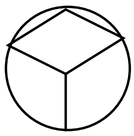
15



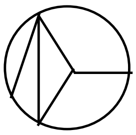
16



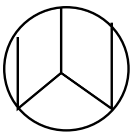
17



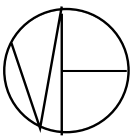
18



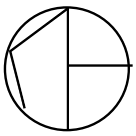
19



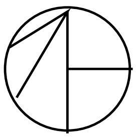
20



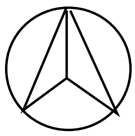
21



22



23

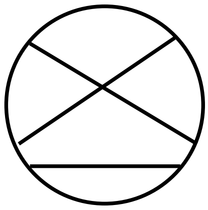


24

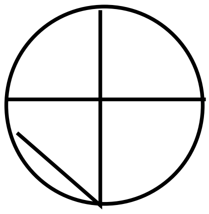
b) Number of internal nodes i = 1 (continued)

ii) degree 4 node and iii) degree 5 node

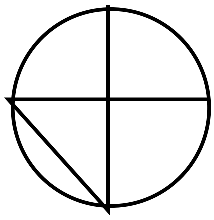
ii)



1

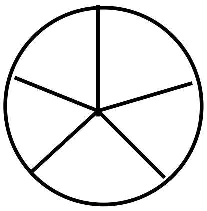


2



3

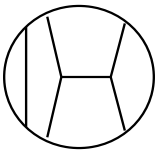
iii)



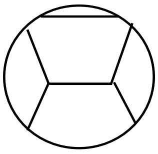
1

c) Number of internal nodes i = 2

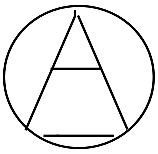
i) two degree 3 nodes



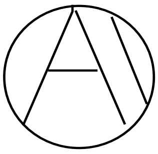
1



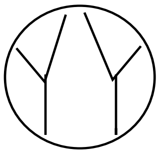
2



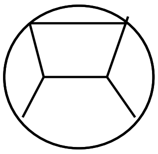
3



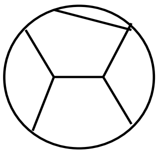
4



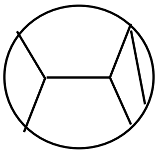
5



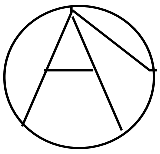
6



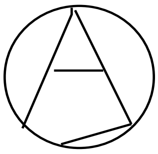
7



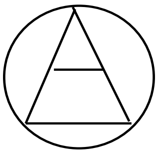
8



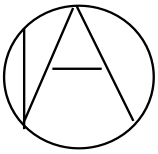
9



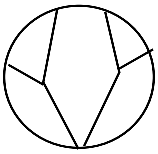
10



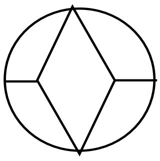
11



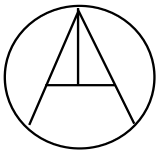
12



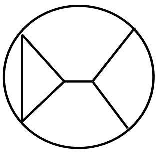
13



14

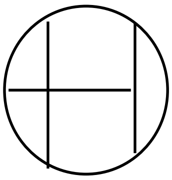


15

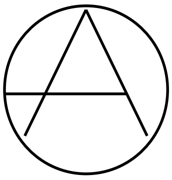


16

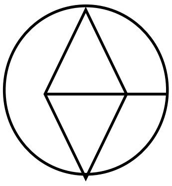
ii) one degree 3 node and one degree 4 node



1



2

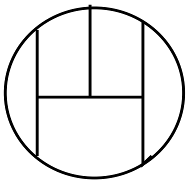


3

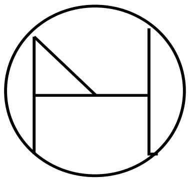
d) **Number of internal nodes i = 3**

i) **three degree 3 nodes and** ii) **two degree 3 nodes and one degree 4 node**

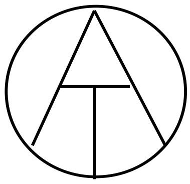
i)



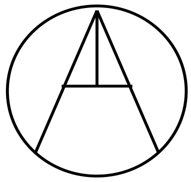
1



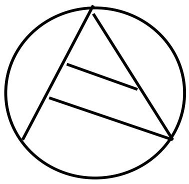
2



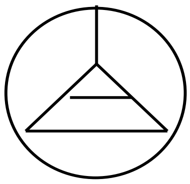
3



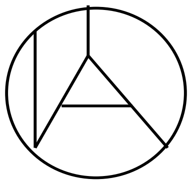
4



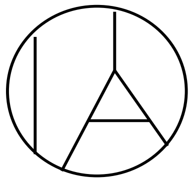
5



6

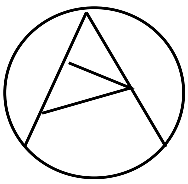


7

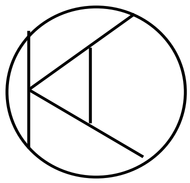


8

ii)



1

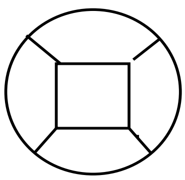


2

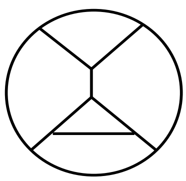
e) **Number of internal nodes i = 4**

i) **four degree 3 nodes and** ii) **three degree 3 nodes and one degree 4 node**

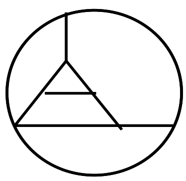
i)



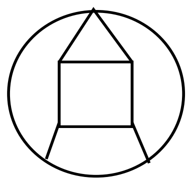
1



2

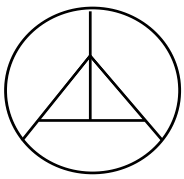


3



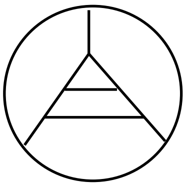
4

ii)



1

f) Number of internal nodes i = 5



1

Total number: $29 + 24 + (3+1) + (16+3) + (8+2) + (4+1) + 1 = 92$.